

Mirror dark matter and galaxy core densities

Rabindra N. Mohapatra

Department of Physics, University of Maryland, College Park, Maryland 20742

Vigdor L. Teplitz

Department of Physics, Southern Methodist University, Dallas, Texas 75275

*and Office of Science and Technology Policy, Executive Office of the President, Washington, DC 20502**

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We present a particle physics realization of a recent suggestion by Spergel and Steinhardt that collisional but dissipationless dark matter may resolve the core density problem in dark-matter-dominated galaxies such as the dwarf galaxies. The realization is the asymmetric mirror universe model introduced to explain the neutrino puzzles and the microlensing anomaly. The mirror baryons are the dark matter particles with the desired properties. The time scales are right for the resolution of the core density problem and the formation of mirror stars (MACHOs observed in microlensing experiments). The mass of the region homogenized by Silk damping is between a dwarf and a large galaxy.

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I. INTRODUCTION

Dark matter constitutes the bulk of the matter in the universe and a proper understanding of the nature of the new particle that plays this role has profound implications not only for cosmology but also for particle physics beyond the standard model [1]. It is therefore not surprising that one of the major areas of research in both particle physics and cosmology continues to be the physics of dark matter.

Apart from the simple requirement that the right particle physics candidate must have properties that yield the requisite relic density and mass to dominate the mass content of the universe, it should be required to provide a satisfactory resolution of three puzzles of dark matter physics: (i) why is it that the contribution of baryons to the mass density (Ω) of the universe is almost of the same order as the contribution of the dark matter to it? (ii) how does one understand the dark objects with mass $\sim 0.5M_\odot$ observed in the Massive Compact Halo Object (MACHO) experiment [2], which are supposed to constitute up to 50% of the mass [3] of the halo of the Milky Way galaxy and presumably be connected to the dark constituent that contributes to Ω (in a manner that satisfies the “environmental impact” conditions of Freese *et al.* [4])? and, finally (iii) what explains the density profile of dark matter in galactic halos—in particular, the apparent evidence in favor of the fact that the core density of galactic halos remain constant as the radius goes to zero?

There are many particle physics candidates for the dark constituent of the universe. Generally speaking, the prime consideration that leads to such candidates is that they yield the right order of magnitude for the relic density and mass necessary to get the desired $\Omega_{DM} \approx 0.2-1$. This is, of course, the minimal criterion for any such candidate and requires that the annihilation cross section of the particles must be in a very specific range correlated with their mass. The most widely discussed candidates are the lightest supersymmetric

particle (LSP) and the Peccei-Quinn particle, the axion. The first is expected to have a mass in the range of 100 GeV whereas the mass of the second would be in the range of $\sim 10^{-6}$ eV. (Compare these values with the proton mass of one GeV.) To understand within these models why $\Omega_B \sim \Omega_{DM}$, one needs to work in a special range for the parameters of the theory. In either of these pictures, the MACHO observations must have a separate explanation. Thus it may not be unfair to say that these two most popular candidates do not resolve the first two of the three dark matter puzzles.

In recent years it has been emphasized that the LSP and the axion may also have difficulty in explaining the observed core density behavior of dwarf spheroidal galaxies which are known to be dark matter dominated. The point here is that both the axions and neutralinos, being collisionless and non-relativistic, accumulate at the core of galactic halos, leading to a core density $\rho(R)$ which goes like R^{-2} rather than a constant which seems to fit data better [5–7]. We will refer to this as the core density puzzle.

This last puzzle has motivated Spergel and Steinhardt [8] to revive and reinvigorate an old idea [9] that dark matter may be strongly self-interacting, which, for the right range of the parameters of the particle, may lead to a halo core which is much less dense and hence in better agreement with observations. To be more specific, it was noted in [8] that if the dark matter particle is self-interacting and has mean free path of collision of about a kpc to a Mpc, then the core on this scale cannot “keep on accumulating” dark matter particle, since these will now scatter and “diffuse out.” For typical dark matter particle densities of order of one particle per cm^3 , this requires a cross section for scattering of $\sigma \approx 10^{-21} - 10^{-24} \text{ cm}^2$. Furthermore, in order to prevent dissipation which would lead to cooling and collapse to the core, one has a lower limit on the mass of the exchanged particle that must exceed typical “virial” energy of particles ($\sim \text{keV}$). An alternative possibility is that the core is optically thick to exchanged particles. If these considerations stand the test of time, a theoretical challenge would be to look for alternative dark matter candidates (different from the popular

*Address until 30 June 2000.

ones described above) and the associated scenarios for physics beyond the standard model.

A class of models known as mirror universe models have recently been discussed. These are motivated theoretically by string theory and experimentally by neutrino physics. These models predict the existence of a mirror sector of the universe with matter and force content identical to the familiar sector (prior to symmetry breaking) [10–12]. Symmetry breaking may either keep the mirror symmetry exact or it may break it. This leads to two classes of mirror models: the symmetric mirror model, where all masses and forces in the two sectors remain the same after symmetry breaking [12] and the asymmetric mirror model [11] where the masses in the mirror sector are larger than those in the familiar sector. The mirror particles interact with the mirror photon and not the familiar photon so that they remain dark to our observations. Since the lightest particles of the mirror sector (other than the neutrinos), the mirror proton and the mirror electron (like in the familiar sector) are stable and will have abundances similar to the familiar protons and electrons, the proton being heavier could certainly qualify as a dark matter candidate. It has indeed been pointed out [14,13] that, consistent with the cosmological constraints of the mirror universe theory, the mirror baryons have the desired relic density to play the role of dark matter of the universe.

The additional neutrinos of the mirror sector are the sterile neutrinos which appear to be needed in order to have a simultaneous understanding of the three different neutrino oscillations, i.e., solar, atmospheric, and the Liquid Scintillation Neutrino Detector (LSND) observations. In fact, one view of neutrino oscillation explanations of these phenomena fixes the ratio of familiar particle mass to the mirror particle mass thereby narrowing down the freedom of the mirror sector parameters. If indeed sterile neutrinos turn out to be required, mirror universe theory is one of the few models where they appear naturally with mass in the desired range. If we denote the mass ratio $m_{p'}/m_p = \zeta$, then a value of $\zeta \sim 10$ is required to explain the neutrino puzzles. What is more interesting is that for the same range of parameters that are required to solve the neutrino puzzles (i.e., $\zeta \sim 10$), mirror matter can also provide an explanation of the microlensing observations [13]—in particular why the observed MACHOs have a mass very near the solar mass and are still dark.

It is the goal of this paper to show that the same mirror universe framework can also explain the core density puzzle of galactic halos. The basic idea is that mirror sector $H' - H'$ (i.e., mirror hydrogen) scattering, with its large geometric cross section, is a natural candidate for strongly interacting dark matter of Ref. [8]. Thus the mirror matter model has the desirable properties that it can naturally explain all three dark matter puzzles. It is worth noting that the asymmetric mirror model was not originally designed for this purpose but rather to explain the neutrino puzzles and indeed it is gratifying that slight modification of the framework (increasing the QCD scale) that solves the neutrino puzzles also solves the dark matter puzzles.

II. MIRROR MATTER AS DARK MATTER

Let us start with a brief overview of the mirror matter models [11,12]. The basic idea of the model is extremely simple: *duplicate the standard model or any extension of it in the gauge symmetric Lagrangian (and allow for the possibility that symmetry breaking may be different in the two sectors)*. There is an exact mirror symmetry connecting the Lagrangians (prior to symmetry breaking) describing physics in each sector. Clearly the W 's, γ 's, etc., of each sector are different from those in the other as are the quarks and leptons. When the symmetry breaking scale is different in the two sectors, we will call this the asymmetric mirror model [11]. The QCD scale being an independent scale in the theory could be arbitrary. We will allow both the weak scale as well as the QCD scale of the mirror sector to be different from that of the familiar sector [13] and assume the same common ratio ζ for both scales, i.e., $\langle H' \rangle / \langle H \rangle = \Lambda' / \Lambda \equiv \zeta$.

It is assumed that the two sectors in the universe are connected by only gravitational interactions. It was shown in [11,12] that gravity induced nonrenormalizable operators generate mixings between the familiar and the mirror neutrinos, which is one of the ingredients in the resolution of neutrino puzzles. It is of course clear that both sectors of the universe are co-located. Together they evolve according to the rules of the usual big bang model except that the cosmic soups in the two sectors may have different temperature. In fact the constraints of big bang nucleosynthesis require that the post inflation reheat temperature in the mirror sector T'_R be slightly lower than that in the familiar sector T_R (define $\beta \equiv T'_R / T_R$) so that the contribution of the light mirror particles such as ν' , γ' , etc., to nucleosynthesis is not too important. This has been called asymmetric inflation and can be implemented in different ways [15]. Present discussions of big bang nucleosynthesis (BBN) can be used to conclude that roughly $\beta^4 \approx 1/6$ is equivalent to $\delta N_\nu \leq 1$.

Before proceeding to any detailed discussion, let us first note the impact of the asymmetry on physical parameters and processes. First it implies that $m_i \rightarrow \zeta m_i$ with $i = n, p, e, W, Z$. This has important implications which have been summarized before [11,16]. For instance, the binding energy of mirror hydrogen is ζ times larger so that recombination in the mirror sector takes place much earlier than in the visible sector. With $\beta \equiv T'_R / T_R$ as above, mirror recombination temperature is $\zeta / \beta T_r$ where T_r is the recombination temperature in the familiar sector. The mirror sector recombination takes place before familiar sector recombination; this means that density inhomogeneities in the mirror sector begin to grow earlier and familiar matter can fall into it later. One can also compute the contribution of mirror baryons to the mass density of the universe as follows:

$$\frac{\Omega_{B'}}{\Omega_B} \approx \beta^3 \zeta. \quad (1)$$

Here we have assumed that baryon to photon ratio in the familiar and the mirror sectors are the same as would be expected since the dynamics are same in both sectors due to mirror symmetry. Equation (1) implies that both the baryonic

and the mirror baryon contribution to Ω are roughly of the same order, as observed. This provides a resolution of the first conceptual puzzle. Furthermore, if we take $\Omega_B \approx 0.05$, then $\Omega_{B'} \approx 0.2$ would require that $\beta = (4/\zeta)^{1/3}$. From this one can calculate the effective δN_ν using the following formula:

$$\delta N_\nu = 3\beta^4 + \frac{4}{7}\beta^4 \left(\frac{11}{4}\right)^{4/3}, \quad (2)$$

where the last factor $(11/4)^{4/3}$ is due to the reheating of the mirror photon gas subsequent to mirror e^+e^- annihilation. For $\zeta = 20$, this implies $\delta N_\nu \approx 0.6$ and it scales with ζ as $\zeta^{-4/3}$. Thus in principle the idea that mirror baryons are dark matter could be tested by more accurate measurements of primordial He^4 , deuterium, and Li^7 abundances which determine δN_ν .

Clearly to satisfy the inflationary constraint of $\Omega_{TOT} = 1$, we need $\Omega_\Lambda \approx 0.7$. These kinds of numbers for cold dark matter density emerge from current type I supernovae observations. It is interesting to note that if one were to require that $\Omega_{CDM} = 1$, the mirror model would require that ζ be much larger (more than 100) which would then create difficulties in understanding both the neutrino data and the microlensing anomalies. Thus mirror baryons satisfy the most requirements to be the cold dark matter of the universe.

III. MIRROR MATTER COLLISION AND CORE DENSITY OF GALAXIES

To discuss further implications of the mirror cold dark matter for structure formation and the nature of the dark halos, we need to know various cross sections. Using the asymmetry factor ζ , it is easy to see that weak cross sections varies as ζ^{-4} , i.e., $\sigma'_W = \sigma_W/\zeta^4$, the Thomson cross section $\sigma'_T = \sigma_T/\zeta^2$. Nuclear cross sections will also be different in the mirror sector due to different values of the QCD scale in the two sectors. For $\Lambda' \approx (10-15)\Lambda$, we would expect $\sigma'_{N'N'} = (\sigma_{NN}/\zeta^2) \times (g_{\pi'N'N'}/g_{\pi NN})^4$ (for fixed values of energy). With these simple rules, assuming the pion nucleon couplings in both sectors to be identical, we find that $\sigma_{N'N'}$ to be of order 10^{-30} cm^2 or so. This is clearly too small to make a difference in the core density problem.

Let us now focus on the atomic forces. We would expect the mirror dark matter to be mostly in the form of atomic hydrogen (H'). In the familiar sector, the hydrogen atomic scattering is of order πa_0^2 , (where a_0 is the Bohr radius) and is of order 10^{-16} cm^2 . Since the Bohr radius scales like ζ^{-1} when we consider the mirror sector, we would expect the collisional mean free path due to atomic scattering to be of order $10^{17}\zeta^3 \text{ cm}$. For $\zeta = 20$, the mean free path is about 0.3 kpc. This is slightly smaller than the lowest allowed value given in Ref. [8]. However, as we argue below, the dangers for lower mean free paths envisioned in Ref. [8], do not apply to our case due to the dynamics of the mirror universe.

As noted in Ref. [8], an additional constraint on the self interacting dark matter arises from the fact that the forces of self interaction must not be such as to allow significant loss of energy from the core of halos since otherwise, core will

lose energy and collapse giving us back the problem we wanted to cure in the first place. Essentially similar reasoning led to the lower bound of 1 kpc on the mean free path in Ref. [8]. In the mirror dark matter model, even though we have mirror photons, there is no dissipation problem. The point is that when there is a collision, it will excite the atom. The atom radiates when it falls back down, but that radiation can be absorbed. Sometimes you ionize, then you get a plasma which also absorbs the radiation. The result would seem to be something like the sun in which it takes a long time for the radiation to get out. In other words, the mirror matter core is optically thick.

In fact, one can estimate the number of collisions a core particle makes since we know the mfp is in the range of 1–0.1 kpc. Taking mfp/virial velocity, we get a time of 10–100 Myr so it makes $10^3 - 10^4$ collisions during the age of the universe. We can also give a crude estimate of the fractional energy loss per particle by using σT^4 for the energy radiated per unit area per unit time, dividing by the number of particles and multiplying by 10^{17} s —the age of the universe, and multiplying by the area of the “core” of radius 1 kpc. This fractional energy loss per particle is negligible enabling us to have a lower mean free path than Ref. [8]. For the case of mirror matter, the core is protected against this collapse by the long time it takes to radiate away energy which is in the form of (mirror) photons that cannot get out [17]. The essential point is the similarity of the mirror sector with the familiar sector, where we know that the photons emitted from the core of a star are very few in number due to minimum ratio of surface to volume and therefore do not lead to collapse of the galaxy cores.

IV. STRUCTURE FORMATION

In this section we will try to make plausible that, in spite of the ζ^2 decrease in cross sections, the facts that (a) structure formation begins earlier in the mirror sector (because recombination occurs before matter-radiation equality) and consequently (b) mirror temperatures are higher, for the same processes, than familiar temperatures, permit the formation of galactic and smaller structures. In doing this, we will make use of our previous work in [16] and [13], as well as that of Tegmark *et al.* [18].

Much of the work of [16] can be carried over to the present work, after suitable modification to take into account the fact that, in the current model, the proton mass scales as ζ . Here, we will assume that primordial perturbations are “curvature” or “adiabatic” perturbations. This means that the scale of the largest structures are set by mirror Silk damping [19]. γ' diffusion wipes out inhomogeneities until the γ' mean free path,

$$\lambda' = [\sigma_T \zeta^{-2} n'_e]^{-1}, \quad (3)$$

where $\zeta^{-2}\sigma_T$ is the mirror matter Thomson cross section and n'_e is its electron number density, becomes greater than one third the horizon distance (ct). Silk damping turns off because the λ' increases as z^{-3} while ct only increases as z^{-2} .

First, we compute, from Silk damping, the masses of the largest structures in this picture. Since structure formation starts with the mirror sector, our assumption is that familiar sector particles will later fall into these. For numerical values below, we will take, $h=0.7$ and $\Omega_{\tilde{B}}=0.2$. We pick $t \sim (z_1/z)^2 s$ with $z_1=4 \times 10^9$ and $n_e^- = \Omega_{\tilde{B}} \rho_c z^3 / (\zeta m_p)$ with $\rho_c = 1.9h^2 \times 10^{-29} \text{ g/cm}^3$. Silk damping stops at around $z_{sd} \sim 8\zeta^3$ which gives

$$\lambda_{sd} \sim 2.5 \times 10^{27} / \zeta^6 \text{ cm}, \quad (4)$$

$$M_{sd} \sim 10^{54} / \zeta^9 \text{ gm}.$$

Note that, for $\zeta \sim 10$, this is about the mass (and size) of a large galaxy. This coincidence could be an important factor in understanding galaxy sizes should this model correspond to reality.

As in [16], we parametrize the separation of M_{sd} from the expanding universe as taking place at

$$z_{stop} = z_M z_{sd}, \quad (5)$$

with

$$R_G = \lambda_{sd} / z_M. \quad (6)$$

After violent relaxation we have for the proton temperature

$$T_p = GM_G \zeta m_p / R_G \sim 10^{-4} z_M / \zeta^2 \text{ ergs}, \quad (7)$$

with ρ , outside the central plateau, given by

$$\rho(R) = A/R^2 \sim 10^{26} z_M / (\zeta^3 R^2) \text{ gm/cm}^3. \quad (8)$$

We now turn to the question of whether this isothermal sphere is likely to fragment and form mirror stars. For this we compute the amount of mirror molecular hydrogen since it is its collisional excitation (and subsequent radiation) that is believed to be the chief mechanism that provides cooling for the formation of stars. If the rate for this mechanism is faster than the rate for free fall into the mass of the structure at issue, we can expect local regions to cool fast enough to result in fragmentation of that structure. We do here a very rough estimate of mirror galaxy fragmentation into mirror globular clusters, using the results of [18], but leave to a more detailed work further fragmentation into the $0.5M_\odot$ structures predicted in [13].

Reference [18] give a useful approximation to their numerical results for the fraction of molecular hydrogen, f_2 (f_0 denotes its primordial value):

$$f_2(t) = f_0 + (k_m/k_1) \ln[1 + x_0 n k_1 t], \quad (9)$$

where, as a first try, k_m can be taken as just the rate for $H + e^- \rightarrow H^- + \gamma$ (which they conveniently give as about $2 \times 10^{-18} T^{.88} \text{ cm}^3 \text{ s}^{-1}$), while k_1 is the rate for $H^+ + e^- \rightarrow H + \gamma$ ($2 \times 10^{-10} T^{-0.64} \text{ cm}^3 \text{ s}^{-1}$). Equation (8) is the result of H_2 production from the catalytic reactions $H + e^- \rightarrow H^- + \gamma$ followed by $H + H^- \rightarrow H_2 + e^-$ competing against the recombination reaction that destroys the catalyst, free electrons, (approximately) as $1/t$ (assuming constant density).

Our goal here is to show from Eq. (8) that it is plausible that f_2 , the fraction of molecular hydrogen, rises from its primordial value of 10^{-6} to the region above 10^{-4} where cooling tends to be competitive with free fall.

First, we note that, if $k = \langle v \sigma \rangle \sim AT^\gamma \text{ cm}^3/\text{s}$, for familiar e and p , we expect that, for mirror e and p , scaling with ζ to go as $\zeta^{-(2+\gamma)} AT^\gamma$, since σ must go as ζ^{-2} and all factors of T must be divided by some combination of m_e and M_p , both of which go as ζ (making this model much easier to compute from than that of [16]).

We now estimate fragmentation. From Eq. (6) we see that the galactic temperature should begin at about 10 eV at a time when the cosmic temperature is about 1 eV and the cosmic gamma number density is about 10^9 cm^3 . The rate for ‘‘compton cooling’’ is very fast at this high density (unlike at later times for the familiar case) and there should be rapid cooling to about 1 eV. We can now compute the Jeans length for fragments as a function of distance R from galaxy central. We use

$$\rho_J = (T/Gm)^3 / M^2. \quad (10)$$

If we set the Jeans mass, M , to $4\pi r^3 \rho_J / 3$, we can solve for r obtaining [if we are careful to convert T in Eq. (6) from ergs]

$$r = R[10^{-7} \zeta^2 / z_M]^{1/2} \sim 10^{-3.5} R. \quad (11)$$

Now inserting into Eq. (8) gives the coefficient of the log term on the order of 10^{-2} and the argument varying from 10^{-13} to 10^{-17} as R varies from 1 to 100 kiloparsecs while the free fall time $[(G\rho)^{-1/2}]$ varies from 10^{14} to 10^{16} . This would appear to indicate the likelihood of fragmentation of the original Silk damping structure into smaller units, (outside the optically thick core) and the eventual formation of the $0.5M_{\text{odot}}$ black holes that explain the microlensing events of [13].

V. CONCLUSION

The asymmetric mirror model [11] was originally proposed to solve the neutrino puzzles. Subsequently, it was advocated [13,14] as providing an alternative dark matter candidate. Then it was shown to have the advantage of resolving the microlensing anomaly in a ‘‘nonpolluting’’ manner [4]. In this paper we have argued that the model could additionally provide an explanation of a fourth problem. It appears to be a realization of the mechanism of Spergel and Steinhardt for understanding the core density profile of galaxies by means of builtin self interactions of mirror matter.

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